Budget Constraints Part One: How To Think About and Model Them

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Abstract

This section covers the basics of writing budget constraints. It covers the concept of a budget constraint, why they matter so much and how to consider the dimensions of the variables so that you can check that your constraint "makes sense". These are really the most basic issues and first steps in budget constraints which are a key component of almost any economic model.

1 Some Motivation

As a grad student and eventually as a professor, you'll very, very often have to write budget constraints. After all, the core to economic analysis is that individuals maximize their utility subject to their budget constraint. We live in a world with unlimited wants and limited resources. The "limited resources" part is expressed mathematically as a budget constraint.

It is also often true that "all the action" is in the constraint. If you think about it, if people are maximizing the same utility function all the time, the real difference in each environment is the budget constraint. For example, I want to consume more. The thing that makes me choose things differently in different environments then is the constraint: how much is the price of A versus B and how many other options are there and what are there prices and how much money does the individual have and can they borrow money from the future or do they have money saved from the past, etc. All those are budget constraint issues. So, it's important to think clearly about budget constraints.

My experience with budget constraints (especially in grad school) was that at first they are difficult until you are comfortable thinking in math terms (i.e., more fluent in this formal language) and they always seem to be different for every problem (because, of course, they are different for the reasons I explained above). Then, they become extremely simple. The good news is that once you are fluent in math-econese, they stay simple for most problems. Then, when you get into your dissertation and later when you write your own papers, they become difficult again because, as I said before, much of the action happens around the budget constraint. All that being said, let's start with the basics.

2 Budget Constraint Basics

At the most basic level, budget constraints are exactly what their name implies "constraints" (on your choices) that are due to your (limited) "budget". If you walk into a store and only have cash in your pocket, you first have to know how much money you have. Suppose it's a fruit stand and you like all the fruit there. You need to know how much money you have and how much each piece of fruit costs before you decide what to buy, or more precisely, how much of which fruit to buy. If your budget was not a constraint on your feasible set of choices (often called your "choice set" or, surprise, your "feasible choice set"), then you could buy an infinite amount of everything.[1]

[1] At a deeper level, this could happen if your money was infinite or if prices were zero, but then your true budget constraint would need to include things like time and how much you can carry.

In the end, you have to have enough money to cover your purchases. That is, your money must be at least as much as you need to buy what you choose. The "at least" means you can have more money than you need or you can have exactly what you need to pay for what you buy, but you can't have less money than you spend. Mathematically,

$$Money \geq Spending$$

Money is pretty easy to represent. We can use just M. But spending has two parts, the price and the quantity. If you bought apples, to know how much you spent, I need to know the price per apple, P_a, and the number of apples you bought, C_a (for quantity "consumed" since we'll assume you consume what you buy, not just buy and stock up on apples). So, if you only bought apples, you'd spend $P_a * C_a$. then, we'd have,

$$M \geq P_a \times C_a.$$

3 Dimensions Part One

Now what if you walked into the store, or up to the fruit stand in this case, pulled the dollars out of your pocket but then noticed that all the prices were in, say, pesos. If M represents USD but P_a and P_b are in pesos, it is meaningless to write $M \geq P_a \times C_a$. This is a first example of why dimensions are important.

But there's actually more. Each variable has a "dimension". For example, the price of something (let's switch back to dollars now) is "dollars per good" or more generally "currency per unit". Think about your own experience: How much is that t-shirt? ... Oh, it's $10.00. Technically, it's $10 per t-shirt, or $10/t-shirt. That's always the case. The price of a car is $40,000 per car. The price of a burger is $2.50 per burger. And so on. So we say "the dimension of price is dollars per unit".

Well then, what is the dimension of C? It's consumption of some good or service. So, if I ate 2 burgers and they were $2.50 each, then the "$2.50 each" is price and the "2 burgers" is the quantity. The easiest way to model, or think about, C, is just as units. The dimension of C is units or "goods" (i.e., it's the "quantity" part.

That leaves the easiest for last: what's the dimension of M? It's literally just money. So, the dimension of the stock of money, M, is dollars.

Now let's return to our equation, $M \geq P_a * C_a$, and rewrite it in dimensions. If we end up with the same dimensions on both sides of the equals, we'll know we're in good shape.

$$\underbrace{M}_{dollars} \geq \underbrace{P_a}_{\frac{dollars}{good}} * \underbrace{C_a}_{goods}$$

$$dollars \geq \frac{dollars}{\cancel{good}} * \cancel{goods}$$

$$dollars \geq dollars$$

$$M \geq \cancel{P_a} * C_a$$

Notice that the dimensions cancel just like anything else. In the end, once everything's canceled out, the dimensions on each side of the equality sign must be the same. When it's not, you've made a mistake somewhere.

Let's do another one. Now suppose there are two goods to choose from, good "a" and good "b". Since we were using a fruit stand example, we could imagine these are apples and bananas. Check the dimensions in the following:

$$M \geq C_a + C_b$$

In dimensions

$$dollars \geq apples + bananas$$

None of the units are the same! So this can't be right! This is how we know we forgot something. In this case, we forgot the prices. Let's write it again, check the dimensions and see what happens now:

$$M \geq P_a * C_a + P_b * C_b$$

In dimensions

$$dollars \geq \frac{dollars}{apple} * apples + \frac{dollars}{banana} * bananas$$

$$dollars \geq \frac{dollars}{\cancel{apple}} * \cancel{apples} + \frac{dollars}{\cancel{banana}} * \cancel{bananas}$$

$$dollars \geq dollars + dollars.$$

Since the dimensions are all the same, we know that at least we got the dimensions right. It could still be a totally wrong constraint, but at least the dimensions are consistent.

Consider one last challenging constraint. Suppose you are thinking about a fruit stand owner who sells domestic apples and internationally purchased bananas. He keeps his money in dollars but the price of apples are in USD and the bananas are in pesos. Try to write the constraint again and check the dimensions.

$$M \geq P_a * C_a + P_b * C_b$$

In dimensions

$$USD \geq \frac{USD}{apple} * apples + \frac{pesos}{banana} * bananas$$

$$USD \geq \frac{USD}{\cancel{apple}} * \cancel{apples} + \frac{pesos}{\cancel{banana}} * \cancel{bananas}$$

$$USD \geq USD + pesos.$$

Uh oh, we didn't get the same dimensions. We have a mistake. Something is missing. Either I need to add something to all the USD-denominated variables to convert everything into pesos or I need to do something to convert the peso-denominated variable to USD. Which one to choose depends on what country you're in and hence which currency you feel comfortable with.

I need something multiplying the P_b that has dimensions that will cancel with the pesos and just leave dollars. So, this time, let's go backwards by starting with dimensions. I need the following:

$$USD \geq \frac{USD}{apple} * apples + \frac{USD}{peso} \frac{pesos}{banana} * bananas$$

sure enough, adding this $\frac{USD}{peso}$ variable will do the job...

$$USD \geq \frac{USD}{\cancel{apple}} * \cancel{apples} + \frac{USD}{\cancel{peso}} \frac{\cancel{pesos}}{\cancel{banana}} * \cancel{bananas}$$

which leaves

$$dollars \geq dollars + dollars$$

.

So, we are good again. Looking at dimensions, you knew what you needed to get all the dimensions the same, but did that variable we added have any economic sense? In this case, yes, it did. It's the exchange rate, E. An exchange rate is a price and the dimensions of all prices are "currency per unit of what is being purchased". In this case the fruit stand owner is using his dollars (the currency) to purchase pesos so the dimensions of E are $\frac{USD}{peso}$. If we assumed that the fruit stand is in Mexico, then he might have made M denominated in pesos and converted the apple price into pesos by using an exchange rate $\frac{pesos}{USD}$. That's a modeling choice and beyond our scope here. But you can see why dimensions matter and also how writing the constraint reflects modeling assumptions. Now let's see another way dimensions matter.

4 Dimensions Part Two: The Numeraire and Real Variables

As in the above international example, it is often the case, even domestically, that you have more than one good. As a result, it is often easier to think in terms of real variables [2] and we can convert "nominal budget constraints" like this $M \geq P_a * C_a$ into a "real budget constraint" by dividing through by P_a

$$\frac{M}{P_a} \geq \frac{P_a * C_a}{P_a}$$

.

As a test to see if you understand the last section, convert that to dimensions and see if you get "all goods" (in this case, all apples). The left hand side is now what we call "the real money supply" and represents the amount of goods you are able to buy with one unit of your cash. The right hand side of the equation now represents "real apple consumption", or just the quantity of apples consumed.

But what about our apple and banana case $M \geq P_a * C_a + P_b * C_b$? Which price should we divide by?

Once again, which one you choose could be completely irrelevant to the economic question or model you are thinking about. In that case, just pick one and divide everything by it. When you have many, many goods, frequently people will just divide through by the first price, P_1, to keep it simple. If there's a specific modeling reason, then you might choose to think of everything in terms of bananas or in terms of apples, so you'd pick P_b or P_a, respectively.

Whichever price you divide by, we call the associate good the **"numeraire"** good [3]. If you divide everything by P_a then we say "apples are acting as the numeraire" and if you divide by P_b then we say "bananas are acting as the numeraire". To practice, you should divide first by P_a and then check the dimensions, then do it again dividing by P_b.

As one last note on using a numeraire to simplify the math, economists will frequently let the numeraire equal 1 so it drops out of all the equations. So they'll say, "in this model we'll let apples be the numeraire and set the price to 1 for simplicity". Since it divides everything, it doesn't really

[2] Recall that real variables are nominal variables divided by a price. It's precisely because of the dimensions involved that the "nominal variable" is converted into a "real variable". We even teach undergrads that nominal variables are still "in dollar terms" and real variables are "in terms of goods". It's a dimension thing.

[3] This is a French word and goes back to the father of mathematical economics, Leon Walras. He was a French economist who first started working on a general theory of equilibrium. He did so by considering many, many goods and prices all at once and hence divided everything through by one of the prices to simplify things and he called it the numeraire good. The name has stuck ever since. I'm told that in French a numeraire is a currency, money or other, that is used to facilitate exchange.

matter what the value is, so you may as well let it be something easy like 1.

BUT, **fair warning!** ... sometimes you forget that the 1 still exists and has the dimensions of P_a. If you use a numeraire and set it to one, then later decide to check your dimensions to make sure you didn't mess something up, unless you remember there's a 1 hiding in your equations with a specific dimension, you will find that the dimensions are off and could spend hours searching for your mistake. Take it from my experience, first leave the numeraire as P_a and don't make it 1 so you have to keep it in your equations through all your manipulations. Then check your dimensions and if it's still off, you know it's not something silly like you forgot there's a numeraire. Many times, however, it was indeed something silly like forgetting the numeraire.

5 Closing Reminder Summary

Let's end this section with a summary of the two key points that will help you:

1. **Gather all the "funds"** and other "plus items" that add to what can be spent on the left hand side and all the expenses on the right hand side. This alone helps keeps things clear in your head generally and specifically when you begin to move this to math.

2. **Check your dimensions**. In the end, the dimensions of every item that is being added up on both sides of the equality (or inequality) sign should be the same. If they are not, something is wrong (i.e., you made a mistake) or you left something out.

These two keys will serve you very well to help keep the basics clear in your head. If you get used to them now, they'll help you save time in solving problems in your first year of grad school and you'll find they are even more valuable in adding clarity and hence simplicity to your life later when you are developing your own models.