Basic Matrix Terms and Definitions

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Abstract

This is very basic material. It collects the most basic terms and definitions: naming elements, matrix dimensions, square matrices, scalars, vectors, transpose, symmetric, idempotent, identity, null, and conformable matrices.

1 Terms and Definitions

Let's start with defining a few matrices that we can add, subtract, multiply, and so on. We'll make matrices **A**, **B**, **C**, **D**, and **E**

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix}, \mathbf{D} = \begin{bmatrix} d_{11} & d_{12} \end{bmatrix}, \mathbf{E} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix}. \tag{1.1}$$

Note: When writing matrix A or B, we generally use bold letters so I will try to use bold for matrices, hence **A** and **B**.

Elements of a Matrix: The order of the objects, or "elements", in matrices is critical and the shape of matrices is critical. So we need terms to refer to both of those critical aspects of matrices. The objects in a matrix are called its "elements" and are referred to by their location in the matrix.

We generally use "i" for rows and "j" for columns so that the elements of the matrix **A** are $a_{i,j}$. This tells us, for example, that $a_{1,2}$ is in the first row, second column (i.e., upper right corner). Or we can be more general and discuss a generic element $a_{i,j}$ or the "i,j-th element" of matrix **A**. This allows us and efficient way to reference say the elements $a_{i,j}$ when $i = j$, which would be the diagonal elements (check matrices **A**, **B** and **E** above to see why) or $a_{i,j}$ when $i \neq j$, which would be the off-diagonal elements and so on.

Dimensions: A matrix has a "dimension" that's defined by the number of its rows and columns, which happens to tell you the matrix's shape as well. When we talk about the matrix as a whole, we think about the total number of rows and the total number of columns. We use "r" for total number rows and "c" for the total number of columns. So a matrix is of dimension "$r \times c$".

Square Matrices: The matrix **A** in (1.1) is a 2x2 matrix as is **B**. Matrix **E** is 3x3. Whenever the number of rows equals the number of columns, $r = c$, we call it a "square matrix".

Scalars: Whenever a "matrix" gets down to both "i" and "j" being 1, then there is only one element. This is no longer technically a matrix and no longer follows the rules for matrix algebra. It is called a **"scalar"**.

Vectors: When either "i" or "j" is one, 1, we call the matrix a "vector". When "i" is 1, then there is only 1 row and it's called a **"row vector"**. Row vectors are of dimension, $1 \times c$. When "j" is 1, then there is just 1 column and it's called a **"column vector"**. Column vectors are of dimension, $r \times 1$.

In our example matrices, (1.1), matrix **C** is a 2×1 column vector and matrix **D** is a 1×2 row vector. And, you can always think of bigger matrices as collections of column and row vectors. Sometimes that's a helpful way to think of them and break them up into their component vectors. For example, **A** could be thought of as two column vectors, $\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$ and $\begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}$ or two row vectors $[a_{11}\ a_{12}]$ and $[a_{21}\ a_{22}]$.

Transpose: When you convert the columns of a matrix into rows to make a new matrix, we say that the second matrix is the **"transpose"** of the first and denote it with "$'$" or T. For example, the transpose of matrix **A** is $\mathbf{A}'$ or $\mathbf{A}^T$. Let

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \mathbf{A}' = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Finally, note that transposing a row vector makes a column vector and vice versa. So, using (1.1),

$$\mathbf{C}^T = [c_{11}\ c_{21}]$$

and

$$\mathbf{D}^T = \begin{bmatrix} d_{11} \\ d_{12} \end{bmatrix}.$$

Symmetric Matrix: Any matrix which is its own transpose is called symmetric, $\mathbf{A} = \mathbf{A}^T$.

Idempotent Matrix: An idempotent matrix is a symmetric matrix for which multiplying it by itself yields itself. That is, the following holds, $\mathbf{A} \times \mathbf{A} = \mathbf{A}$.

Identity Matrix: The identity matrix is a matrix with ones along the diagonal and zeros everywhere else.

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In the world of matrices, it functions like a "1" in normal algebra. In particular, any matrix times an identity matrix is itself, $\mathbf{AI} = \mathbf{IA} = \mathbf{A}$. That includes the identity matrix itself, $\mathbf{I} \times \mathbf{I} = \mathbf{I}^2 = \mathbf{I}$.

Finally, the identity matrix is symmetric and idempotent.

Null Matrix: The null matrix is a matrix that has only zeros in it. It functions like a zero in normal, scalar, algebra.

$$\mathbf{0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Note that it can be any shape or size. It can be a column or row vector, for example.

Conformable Matrices: Two matrices are said to be "conformable" if their dimensions are such that the number of columns in the first matrix equal the number of rows in the second matrix. That is, if **A** is an $r_a \times c_a$ matrix and **B** is an $r_b \times c_b$ matrix, then **A** and **B** are conformable if $c_a = r_b$. Basically **A** is as long as **B** is tall. Conformability is important because two matrices can only be multiplied together if they are conformable.