Exchange Rates

Chris Ball

This version: March 6, 2018

Abstract

This material covers the basics of what you need to know to understand exchange rates and exchange rate determination. To get there we also need to cover the dimensions of variables, "hat algebra" and trade in two markets to get interest parity. This was written for my Open Economy Macroeconomics course at Quinnipiac University.

1 The dimensions of variables

The dimension of a variable is it's measurement or unit. In physics this would be things like "feet" or "pounds per square meter" and so on. In economics it's things like "dollars" or "dollars per hour".

For example, the dimensions of a price are "dollars per good". If you walk in a store and ask "how much is that shirt?" they say, for example "20 dollars" by which they clearly mean "20 dollars per shirt". The dimensions of a wage are "dollars per hour worked". And so on.

The reason this becomes important in an international context is that different currencies are used in different places around the world. Notice that it's easiest to imagine each country having it's own, different and distinct currency, but it's not a pure mapping of 1 currency per country. The Eurozone includes mulitple countries using the same currency, the Euro. Similarly there are countries that have "dollarized" and just adopted the US dollar. In other cases, there might be some markets in boarder towns that use multiple currencies. Or specific global markets, like the crude oil market, may be in US dollars even though it's a global market. But, you can think of different currencies in different countries and it's a good place to start.

Let's get a little more technical and see some examples.

If the price level [1] is $P = \frac{\$}{goods}$ and real GDP is $y = \frac{goods}{time}$, then $Py = \left(\frac{\$}{goods}\right)\left(\frac{goods}{time}\right) = \frac{\$}{time}$. This is just nominal GDP, $Y = Py$ which is indeed defined more or less as as "the market value of all goods and services in a year". That is, "dollars per year".

It also allows us to go from nominal variables (i.e., variables still defined in dollar terms) to real variables (i.e., variables just expressing goods and services). In the next example we'll go from nominal wages to real wages.

Again, let the price level be $P = \frac{\$}{good}$. If the nominal wage is $W = \frac{\$}{hour}$ then to find the real wage which should tell us not how many USD you earned per hour of work but how many goods and services you can buy with your earnings per hour of work, we need to divide the nominal wage by the price level, $w = \frac{W}{P}$ and if we consider the dimensions then $w = \frac{W}{P} = \left(\frac{\$}{hour}\right)\left(\frac{goods}{\$}\right) = \frac{goods}{hour}$.

[1] Technically I should write "he dimensions of the price level are..." but it's too cumbersome, so I'm a little sloppy technically but I think more readable in writing it this way.

2 Exchange rates...just another price in the economy

Now consider the problem of figuring out how many hours you have to work in the US to buy 1 dinner in France. The problem is that you have to pay Euros to buy the dinner but you are paid in USD for your work in, say, New York.

The basic problem here is to convert a Euro price, call it P^* whose dimensions are $\frac{Euro}{good}$ into an American price that is $\frac{USD}{good}$. Notice that to be fully comparable the good should be the same. In our example, they are. We are asking "if the Paris restaurant suddenly starting allowing you to pay with US Dollars, how much would it charge? so the mean (i.e., the good) is identical. This will be relevant below (see "The Law of One Price") So we face the following basic problem

$$P^* \times ? = P$$

where the "*" indicates "foreign" so that this reads "the foreign currency price of dinner times something equals the domestic price of dinner". And, in dimension terms

$$\frac{Euro}{dinner} \times \frac{?}{?} = \frac{USD}{dinner}$$

It's pretty obvious that we need something to replace Euros with dollars in order to make the dimensions the same on both the left and the right sides of the equation.

$$\frac{\text{\sout{Euro}}}{dinner} \times \frac{USD}{\text{\sout{Euro}}} = \frac{USD}{dinner}$$

Going back to variables, this tells us that the "?" we had in $P^* \times ? = P$ is a variable with dimensions $\frac{USD}{Euro}$. We call that variable a "Nominal Exchange Rate" and use the symbol "E" for it. So,

$$P^* \times E = P$$

2.1 An exchange rate is just another price

The first thing to notice about the nominal exchange rate is that it's "just another price" in the economy. Prices in the USA are "dollars per good". So, a Toyota might be "30,000 dollars per car" or the medium macchiato I like to buy at Dunkin Donuts is "3.39 dollars per cup" and so on. Well, the exchange rate is just "dollars per Euro". The exchange rate today, March 3, 2018, happens to be "1.23 dollar per Euro".

The exchange rate is just another price in every way possible. It is "local currency per something" or more specifically "local currency per 1 unit of foreign currency" the same way other prices are "local currency per 1 unit of something". As such, note that it's also "in dollar terms" which means it is a "nominal variable" in the economy.

2.2 An exchange rate is bilateral

Since the exchange rate is the price of something - foreign currency - it is as meaningless to talk about "the exchange rate" as it is to talk about "the price"..."the price of what?" The price of computers has fallen over the years. The price of housing has risen in recent years. In the same way it's important to remember that an exchange rate is the US dollar price of a single foreign currency. That is, it is bilateral.

The US exchange rate with the Eurozone (i.e., USD per Euro) can go up while the US exchange rate with Mexico (USD per Peso) goes down and so on in the same way that the US dollar price of eggs can go up while the US dollar price of milk goes down, all in the same grocery store.

In practice, if an economist wants to look at a country's exchange rate with respect to several countries' currencies that it trades with, we might compute a representative basket of goods reflecting goods and services from multiple trading partners and then calculate the USD price of each of their currencies (i.e., the exchange rate with each of the other country's currency) and this is called an "effective exchange rate" or may have other names. We do the same with prices when we consider a typical basket of goods consumers buy and calculate a chain-weighed index of all those prices. In that case we call it "the consumer price index".

Similarly again, we'll sometimes loosely talk about "prices in an economy" or the price level and we'll also talk of "the exchange rate" or "exchange rates" and be general. We do the same in theory when we talk about "the interest rate" in economics when in reality there are thousands of different interest rates.

2.3 Flipping it on it's head...the value of a currency

If I were to tell you that the price of steel is rising, you would understand that steel is becoming more valuable. Notice that in terms of dimension, "steel" is on the denominator when I discuss the price of steel:

$$P_{steel} = \frac{USD}{steel}$$

So if $\uparrow P_{steel}$ implies $\uparrow$ value of steel it means that each dollar can buy less and less steel. In other words, a higher price of steel also means that dollars (at least in terms of how much steel they purchase) are less valuable.

Well, an exchange rate is a price, so it works the same. If we say in the USA that the price of Euros went up, that means the exchange rate with Europe went up (i.e., USD per Euro) and it means that each dollar can buy fewer and fewer Euro. In other words, a higher price of Euros also means that dollars (at least in terms of how many Euros they purchase) are less valuable.

Therefore, it's important to always remember that

1. a **higher exchange rate** is also a **depreciation** in the value of our domestic currency in terms of foreign currency, and
2. a **lower exchange rate** is also an **appreciation** in the value of our domestic currency in terms of foreign currency.

Think about it. A depreciation of your currency means it buys less and less stuff. That means the prices of stuff is going up and up. Same thing for exchange rates. I find that we have to remind ourselves of this over and over. One of the most common mistakes students, financial writers, and others make is to confuse an exchange rate going up with the domestic currency increasing in value (i.e., appreciating). Some financial writers will say instead that "the US dollar rose" which is technically incorrect but at least captures the rise in USD value and is a decline in the exchange rate with respect to another currency. So they not only were a little loose here but they didn't even say ""with respect to what". They probably mean "most major trading partners' currencies".

3 The Law of One Price

In the motivating example of buying dinner in Paris, we noted that the prices are equal (but in different currencies) if the good is 100% identical. In theory this is always true assuming freedom to trade and zero transaction costs (such as transportation costs). That means, the price of a cup of Dunkin Donuts coffee in Hamden, CT is the same as it is in New Haven, CT. The same would be true of any good that is perfectly identical.

In theory then it should also be true across countries. So the price of a cup of Dunkin Donuts coffee should be the same in Hamden, CT and in Budapest, Hungary...if they had Dunkin Donuts in Budapest. But the point is clear.

The reasons are many fold but mostly this is driven by arbitrage. So, if a car is more expensive in NY than in CT, then NY residents will drive to CT to buy their cars (note transaction costs aren't zero, but they are low). This alone will drive up the price in CT and down the price in NY. While people don't usually do this for cups of coffee as in my example, they do do it for alcohol or cigarettes and other goods if they live close enough to the state line.

So, in theory and assuming zero restrictions on the freedom to trade and assuming zero transportation costs and assuming goods are identically the same, then our dinner example holds for all goods and is called "**The Law of One Price**" and in an international context it means

$$P^*E = P$$

where the "*" indicates "foreign". So this says, the foreign price of a good times the exchange rate (i.e., once converted into domestic currency terms) equals the domestic price.

4 Absolute Purchasing Power Parity and Price Levels

Continuing in our perfectly clean theoretical world of no restrictions, no transactions costs and identical goods, the law of one price means we can generalize if we add two more assumptions. Suppose all the exact same goods and services exist in each country and that consumers consume those goods in the same proportions in each country. That all the goods are the same is easy to imagine. The second assumption is that if consumers in the US spend 60% of their income on housing, 20% on clothing and food, and 20% on all other goods and services, then we are assuming that in France or Germany or whatever country we are considering, their residents consume in the exact same proportions. If they do - and we'll assume for now that they do - then it's easy to see that there must be something like a law of one price that also holds for price levels across countries. That is,

$$P^*E = P$$

but now P^* and P represent the price levels in each country. It works because price levels are just indices of prices for baskets of goods and services consumed by local consumers. Since we've assumed all the goods and services are the same and we assumed consumers have the same tastes and preferences and therefore consume the same bundles or baskets of goods in each country, then the value of the indices must be identical too once converted into the same currency. The exchange rate is the exactly same, by the way, since it was just the price of Euros.

Notice that we can also rearrange this as the ratio of price levels,

$$E = \frac{P}{P^*}.$$

Either version of this relationship, $P^*E = P$ and $E = \frac{P}{P^*}$, are called "Absolute Purchasing Power Parity (PPP)" since it is in "absolute" levels and a price index is exactly constructed to indicate a consumer's (and a currency's) purchasing power in terms of the domestic goods and services that country's consumers buy. "Parity" means equal. So, Absolute PPP just says that the purchasing power has to be equal across two countries.

5 Relative Purchasing Power Parity and Inflation

Obviously, Absolute PPP is a bit extreme. And the assumptions behind it don't hold in reality. For starters there are transaction/transportation costs to buying something in France and shipping it to the USA. Secondly while some goods are the same, many goods and especially services differ in the US and France. Thirdly there are restrictions to people being able to freely trade with each other (I can't just buy French wine and start selling it in Connecticut without paying import duties, buying a license in Connecticut, etc.). So it should come as no surprise that in practice, exact, absolute PPP doesn't hold perfectly.

It does seem to hold on average over time, however. That is, when you look at the average change in price levels in two countries over time and the average change in the exchange rate over time, it more or less holds. Again, it's not perfect because the real world isn't perfect, but the differences wash out a lot more when you look at averages over time and as a result the "imperfections" or deviations from our perfect theory wash out too, so the results hold a bit better.

Converting Absolute PPP to percentage changes over time (%Δ) generates the following

$$E = \frac{P}{P^*} \rightarrow \%\Delta E = \%\Delta P - \%\Delta P^*.$$

Since these are price levels and the %Δ in a price level over time is called inflation, this says that the %Δ in the exchange rate over time is equal to domestic inflation minus foreign inflation. Using at“”hat" to denote %Δ and the Greek letter 'π" for inflation, we get

$$\hat{E} = \pi - \pi^*.$$

this formulation is called "**Relative Purchasing Power Parity (PPP)**". It holds better empirically over time and makes a lot of intuitive sense once you stop and think about it for a minute. Recall that an increase in the exchange rate is a depreciation in the value of our currency. While that is initially a little hard to grasp, when we think in terms of inflation, it's more comfortable.

Everyone knows that inflation "erodes the value of a currency". Inflation means "all prices are rising" and hence each dollar (in the USA) buys less and less. So here it's clear that rising prices mean a depreciation of the US dollar. Since the exchange rate is just one of those prices, it's no surprise that it also indicates a depreciation of the US dollar.

Furthermore, Relative PPP makes sense in this context too. Since exchange rates are bilateral (i.e., USD to 1 other currency) and since domestic inflation represents how quickly we erode the value of our domestic currency and foreign inflation represents how quickly foreigners are eroding the value of their currency, then Relative PPP just tells us that the exchange rate rising or falling overtime represents which country is eroding their currency faster. If $\hat{E} > 0$ then domestic inflation is higher than foreign inflation (i.e., $\pi > \pi^*$) so the domestic economy is eroding it's currency's value faster than the foreign economy and hence the domestic currency's value is depreciating relative to the foreign currency. And if $\hat{E} < 0$ then domestic inflation is lower than foreign inflation (i.e., $\pi < \pi^*$) so the domestic economy is eroding it's currency's value more slowly than the foreign economy and hence the domestic currency's value is appreciating relative to the foreign currency. This should make much more clear and intuitive that an **increase in the exchange rate** is a **depreciation** in the domestic currency's value and a **decrease in the exchange rate** is an **appreciation**.

One final note of caution! In practice you must be sure that when you read about an exchange rate, it's been defined as "domestic currency divided by foreign currency". Sometimes people in different countries will define it differently and then everything we just did is upside down. For example in the UK they tend to define it upside down. That means some global reports do this if they are produced in the UK or by UK authors. Also *The Economist* magazine often does this since it's a UK magazine. I personally find it easiest to just check the footnote to see how the publication defines the exchange rate (they usually say it, especially if it's different like in the UK) and then just mentally flip everything they do so it makes sense in the terms we learned it.

6 A first link between monetary theory and the exchange rate

Milton Friedman, the Nobel prize winning monetary theorist, is famous for saying "inflation is everywhere and always a monetary phenomenon". What he meant by that is that, in the end, if you print more and more money, you'll get inflation. And therefore also, if your country has more and more inflation, someone must be printing more and more money. In principles of economics classes, it's often taught as "printing money just means more and more dollars chasing the same goods so prices eventually have to rise". It's impossible for an economy to have more and more persistent inflation over the long run if there isn't more and more money. And in most modern countries the government, in particular the central bank, prints the money[2], then ultimately inflation is a result of an economy's central bank's monetary policy.

6.1 The Quantity Equation of Money

The connection between the money supply which is determined by the central bank's monetary policy and inflation comes through the equation connecting the quantity of money in an economy with the total value of all goods and services sold in an economy, nominal GDP. That equation, because it uses the quantity of money, is called "**The Quantity Equation**":

$$MV \equiv Py$$

where M is the quantity of money (or technically the stock of nominal money supplied), V is the velocity of money, P is the price level, and y is real GDP. The last part, Py is easiest. Real GDP is the quantity of goods and services sold in an economy and the price level is the price of all those goods and services so the two together are "nominal GDP" which is the "value" of goods and services sold in an economy. The nominal stock of money is also easy. It's just the total amount of money in the economy. The confusing piece is "velocity" which is the average times a dollar (in the US) must change hands in order for the stock of money to have generated the nominal GDP in an economy.

I've found this easiest with an example. Suppose our whole economy is just a coffee economy where real GDP is just 100 cups of coffee a year and the price level is just the price of a cup of coffee, let's say \$1.00 per cup of coffee. Then nominal GDP is $Py = \$1.00 \times 100 = \$100$ per year.

Now suppose you count all the dollar bills in the (US) economy. (For simplicity we'll assume all money is just dollar bills in this economy. In reality we count bank deposits and other stuff too, but we're trying to keep it simple here and conceptually adding more complexity doesn't matter at all.) You learn that over this year there was only 1 one-dollar bill in the economy (i.e., $M = 1$)! How can that be?

Well, if there's only 1 one-dollar bill in the economy but \$100 in total sales (i.e., nominal GDP) that since dollar must have changed hands 100 times. You can imagine that the Mr. A buys a cup of coffee from Mr. B for a dollar, now Mr. B has the dollar and buys coffee from Mr.C for a dollar, and so 100 times until 100 cups of coffee were sold. That's all velocity is,

$$V \equiv \frac{Py}{M}.$$

[2]In the US the central bank, the Federal Reserve, technically determines the amount of money to be printed and the Treasury Department does the actual printing, but it's a distinction that's splitting hairs for our purposes.

6.2 The Short Run

In general we assume velocity is constant. Once you do this, you've converted an identity, something always true by definition (hence the $\equiv$ sign), into a theory. We can worry about this detail elsewhere, for now, assume velocity is constant.

Another common assumption in macroeconomics is that the price level (not each price, but their aggregate index, P) is slow to adjust. We call this the "sluggish price" assumption or "sticky price" assumption.

Those two assumptions mean that any change in M, which is exogenous and decided by a central bank, must show up in real GDP, y. So, in the short run,

$$\uparrow M \rightarrow \uparrow y$$

prices may have risen a little, but mostly the increase feeds into y. In general then, in the short run,

$$\%\Delta M = \%\Delta y$$

.

6.3 The Long Run

In the long run however two things come into play. First, the price level is slow to adjust but it does adjust over time. And, second, all we did was print money. Nothing fundamental changed our capital, labor or technology in the economy and those are the things that matter in the long run for real GDP, so the temporary bump in real GDP was "fake" and unsustainable. It's in this sense we often teach that printing money eventually just means more dollars chasing the same goods and hence must increase prices.

So, in the long run, real GDP is unaffected by money (economists call this "Money Neutrality" since "money's effect on real variables is neutral in the long run") but prices fully adjust. Actually that's a macroeconomist's definition of the long run, "after prices have all finished adjusting". So, in the long run we have,

$$\%\Delta M = \%\Delta P$$

or

$$\%\Delta M = \pi$$

and note we often write $\%\Delta M$ as $\hat{M}$. And this links monetary policy to inflation rates and relative PPP links it to exchange rates over time.

6.4 Summarizing the linkage

To summarize, in the short run,

$$\hat{M} = \hat{y}$$

but in the long run,

$$\hat{M} = \pi$$

And

$$\hat{E} = \pi - \pi^*$$

So, all else equal, in the long run, $\hat{M} = \hat{E}$.